DECISION METHODS

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This document explains the decision methods implemented in the mDSS5, a generic decision support system developed to assist the implementation of the Water Framework Directive and the development of the River Basin Plans.

This guide was updated for the mDSS version 5, released in September 2010. Apart of this document, the users may want to consult the Users' guide which explains how the software can be used. Furthermore, a practical tutorial is available.

The software and further resources are available from: http://www.netsymod.eu/mdss/

"The world moves into the future as a result of decisions, not as a result of plans. Plans are significant only insofar as they affect decisions ... if planning is not part of a decision making process, it is a bag of wind, a piece of paper, and worthless diagrams."

[Boulding 1974]

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1 INTRODUCTION

This document explains the multiple criteria decision methods implemented in the mDSS5, a generic DSS developed to assist water authorities in the management of water resources. The software was originally developed in the context of the project MULINO (MULti-sectoral, INtegrated and Operational Decision Support System for Sustainable Use of Water Resources at the Catchment Scale) and further developed and applied with a contribution of several other projects, including DSS-GUIDE, TRANSCAT, NOSTRUM-DSS, NEWATER and BRAHMATWIN.

The mDSS5 is particularly useful when applied together with other tools such as stakeholders' analysis or problem structuring methods. We apply the software a broader analytical framework which we refer to as NetSyMod. This framework is the result of several years of research in the field of environmental evaluations and decision making carried out at FEEM within the Natural Resources Management Research Programme. It consists of a suite of tools aimed at facilitating the involvement of stakeholders and experts in environmental decision making processes.

The main components of the framework are:

• Identification of all potential stakeholders/experts affected by the policy decision under examination. The process proposed to fulfil this objective is a simple approach based on the organisation of brainstorming meetings and on the use of a snowball techniques.

• Social Network Analysis (SNA) which aims at assessing the reciprocal relationships among actors within their local social networks. Characterising the power structure prevalent in the selected group of actors, SNA should ensure that the participatory modelling and/or planning process is not hijacked by powerful groups, but rather, it is truly representative of the whole sample – and population – of interested parties.

• Creative System Modelling (CSM) which provides means for facilitating the process of participatory modelling and, more specifically, for eliciting knowledge and preferences from actors. The key actors chosen in the previous steps of the NetSyMod approach will take part in a participatory workshop during which Cognitive Mapping techniques most suitable for the specific case will be applied.

• Analysis of Options, in which the participatory approach is brought in the field of decision support envisaging the use of specific computer support tools.

In the subsequent section of this chapter these components are briefly described, for an in-depth description see Giupponi et al. (2007). To continue with the mDSS5 decision methods go to the chapter 2.

1.1 ACTORS' ANALYSIS

This initial phases identifies all potential stakeholders/experts involved or affected by the decision under investigation, and singles out those who should take active part in the decision making process.

First of all, it is necessary to identify all potential stakeholders/experts involved in, or affected by, the decision to be undertaken. Within the NetSyMoD framework, a task force group is set up for this purpose, which, through a combination of brainstorming meetings and a modified snowball sampling technique, carries out this task.

When all the relevant actors have been identified, a Social Network Analysis is undertaken, with the aim of assessing the reciprocal relationship among actors. Through the use of questionnaires and interviews, the SNA will allow the identification of key actors, the assessment the power structure among the actors, and the characterisation of their role and position with respect to the decision to be taken.

SNA ensures that the participatory modelling and/or planning process is not hijacked by powerful groups, but rather it is truly representative of the whole spectrum of interests and positions. There are thus three main outputs from the SNA phase, which will be an input into the preparatory phase for the CSM workshop.

1. A list of key stakeholders/experts to be involved in the next phases of NetSyMoD. This will limit the number of participants to a manageable size, and ensure that no important actors are left out of the exercise.

2. The analysis of power will highlight potentially problematic actors and relations, whom the facilitator will need to actively manage during the creative system modelling workshop.

3. A conflict analysis on the basis of position and roles of actors within the network, with the purpose of identifying key alleys and/or opponents, and actors who are opinion setters.

1.2 PROBLEM ANALYSIS

In this phase the problem (or conflict) at hand is scrutinised from various perspectives and viewpoints. The environment in which the problem is embedded is explored and the relevant factors identified.

The problems faced by natural resource managers are complex and their drivers interwoven. It is necessary to identify the most relevant aspects, by focusing on which the major changes can be attained. The exploration of the problem include analyses of legal and institutional frameworks, as well as the economy on various spatial levels and the state of environment. Future development of main drivers and pressures are simulated using models under alternative scenarios.

Different stakeholders (identified in previous step – Actor analysis) hold different perceptions and beliefs about what are the causes of the problem or how it should be tackled. Different techniques have been developed to surface tacit knowledge and deeply held beliefs, including conflict assessment, problem structuring methods, discourse analysis. The individual perspectives are further elaborated in the next step (Creative system modelling) to facilitate collective learning and shared (agreed) boundaries of the problem.

The problem analysis phase typically ends with

1. A list of most relevant drivers governing the perception of the problem at hand

2. Sketch of cause-effect relations between various drivers, identified and explored using multiple methods and models

3. A set of scenarios regarding the future development of the main drivers and cause-effect relations

4. A extensive list of indicators against which the performance of policy measures should be measured

1.3 CREATIVE SYSTEM MODELLING

A shared model of reality is needed for the correct evaluation of policy options. Creative System Modelling (CSM) techniques facilitate the process of participatory modelling and elicitation of knowledge and preferences from actors, thus building a common understanding of the problem.

The key actors identified in the previous step will take part in a participatory workshop, during which cognitive mapping techniques (such as the Hodgson’s hexagon method or a revised Delphi technique) will be used to develop a shared model of the decision problem.

The CSM workshop can have two main aims, depending on the case at hand:

i. building a shared model of the problem, based on cause-effects chains and using the DPSIR conceptual model (Driving Force, Pressure, State, Impact, Response); or

ii. developing shared scenarios, depicting the potential evolutions of the system over time, or under different policies.

The CSM will also serve the purpose of identifying shared evaluation criteria, and eliciting individual and group weights, necessary for the evaluation of policy options through multicriteria analysis.

Creative system modelling provides not only a common ground for the mutual understanding among the parties involved, but also a scientifically sound basis for the development of effective decision support systems (DSSs).

The cognitive map of the decision problem, or the related scenarios, will be the basis for the analysis of options:

- the shared mental maps elicited at the CSM workshop will be the underlying modelling framework for tailoring mDSS to the specific needs;
- the workshop will provide qualitative and/or quantitative indicators to be used in the choice phase in mDSS;
- the workshop may also lead to a quantitative assessment of these indicators, in addition to their identification.

1.4 DSS DESIGN

In this phase numerous tools and information (knowledge) produced in previous steps are assembled into a toolbox or framework. This is necessary to manage the information flow between various process phases, including exchange, transformation, integration, validation and documentation of gathered knowledge.

Many of the previous analyses employ computer-based tools such as databases (and data management systems), visualisation components, and simulation models. Different tools are frequently assembled into a comprehensive Decision Support Systems, normally employing various interconnected and adapted components, controlled by an user interface. This phase address all activities related to the development of interoperable and useable software components and collection of well documented and easily exchangeable data sets (including spatial data and time series):

1. Seamless data flow between various tools and software component
2. User interface which guides user though various stages of the NetSyMod process
3. Quality assurance regarding the integration of different components
4. Documentation and report facilities which explain the process and facilitate the interpretation of results

1.5 POLICY EVALUATION

Policy evaluation consists of choosing one (or more) policy measure from a set of mutually exclusive alternatives, or producing their complete ranking. Numerous methods and techniques have been developed in decision theory to make explicit (transparent) value judgements and assess the extent to which different policies contributed to achieve the pursued goals and objectives.

Decision models (DM) result from the systematic exploration and negotiation of a ‘problem’, including its existence, boundaries and structure. DM comprise alternative courses of actions (policies or policy measures); decision goals - translated into more tangible evaluation criteria - against which the policies are weighed; and preferences, which describe how well the policies satisfy the objectives.

There are normally several candidate policies; for example, high nitrate pollution can be tackled by introducing financial incentives, changing nutrient management in farms, by protecting littoral vegetation and favouring phytodepuration, or by improving the effectiveness of waste water treatment plants, WWTP). Binary (yes/no) choices, such as whether to adhere to the Kyoto protocol for reducing greenhouse gas emissions are frequently indicative of escalating conflicts due to incommensurable ethical principles, values and interests. Goals may refer to competing targets, e.g. macro-economic developments vs. social impact; favouring different policies so that no single option outperforms all others. In these situations, decision makers may be a priory uncertain (undecided) about what policy action is most appropriate. This indecisiveness is a result of the diversity of decision outcomes, which are not uniformly distributed in space and time (e.g. different policy impacts on upstream vs. downstream water users; WWTP extensions may have an earlier impact on nitrate concentration than land use changes) or the values attached to them. Uncertainty in the outcomes of a choice poses yet another challenge for decision making.

The trade-offs or preferences are value judgements, which are frequently not observable and must be revealed or approximated. Such uncovered preferences are context specific and depend on the description and framing of a problem, and how the questions are formulated. For example, to assess the environmental costs of irrigation, one must consider the value of wetlands and riverine ecosystems deprived by water abstraction. These values, regardless of whether they are in monetary terms or relative utility, may be difficult to approximate as the results depend on the respondents’ prior knowledge or on what they think others would approve. In situations involving uncertainty, preferences are formed over probabilities of possible outcomes of the policies and integrated into the decision model. These preferences embody attitudes towards risk (risk aversion vs. risk seeking vs. risk neutrality), defined according to the value individuals attach to the uncertain outcomes of a decision.

Decision methods help to avoid inconsistencies underlying judgement and choice, and make decisions more compatible with normative axioms of rationality. Furthermore, if combined with deliberative techniques, decision methods render policy processes transparent and informed the perspectives or viewpoints of all actors. This is translated into a higher acceptance of the policies.

2 DEFINITIONS

A ***decision problem*** is considered to exist, when a planner or a decision maker (DM) perceives a discrepancy between the current and desired states of a system, and when (i) the DM has alternative courses of action available; (ii) the choice of action can have a significant effect on this perceived difference; and (iii) DM is motivated to make a decision, but he is uncertain a-priori as to which option should be selected. ***Multicriteria decision aid*** (MCA) is a branch of decision theory which deals with decision problems characterised by a number of evaluation criteria. Two fundamental parts of MCA are (i) *Multiple Attribute Decision Making* (MADM), and (ii) *Multiple Objective Decision Making* (MODM). The former approach requires that the choice (selection) be made among a limited number of options which are described by their attributes. The second approach allows the number of potential options to be nearly indefinite. The options are not defined explicitly. Instead, MODM provides a mathematical framework for designing a set of decision alternatives.

Preference is a decision maker's notion about the available options.

Options[1] represent the different choices of action available to the decision maker. *Feasible options* must fulfil the satisfaction level (constrains) given by the decision maker for a set of criteria. A *non-dominated option* refers to the one that is, at least equal in all criterion scores and at least is better in one criterion than other (dominated) options.

A ***criterion*** is a standard of judgement to test the desirability of an option. In MCA the concept of a criterion includes both *attributes* and *objectives* referring to two main directions: *multi-attribute* and *multi-objective* decision-making. An *attribute* is a qualitative or quantitative property that is measurable and relevant to a decision. An *objective* is a statement about the desired state. Functionally related to the attributes, the objectives indicate the directions of improvement (e.g. an objective may be formulated as: minimising the water pollution).

Decision matrix is a (M x N) matrix in which the element x_{ij} indicates the performance of the option a_i evaluated in the terms of the decision criterion c_j. The "raw" performances expressed in different non comparable units and scales are represented in the so called *analysis* matrix. The relative performance (u_{ij}) is constituted by the preference mapping using a value/utility function and expressed in the same scale as the *evaluation* matrix.

Value function (u) is a mathematical representation of human judgements. It translates the performances of the options into value scores, which represent the degree to which a decision objective is matched. I.e.:

$$u(a) > u(b) \Leftrightarrow a \succ b$$
$$u(a) < u(b) \Leftrightarrow a \prec b$$
$$u(a) = u(b) \Leftrightarrow a \sim b$$

where a, b …options
u() …value function
$\succ$ … is prefered
~ …is indifferent

[1] In this document the term "option" is used where many theorists would use the term "alternative".

3 MULTICRITERIA DECISION ANALYSIS - AN INTRODUCTION

3.1 BASIC STEPS OF MCA

Figure 1 shows the basic steps of multicriteria decision analysis as implemented in the mDSS. The decision process starts with problem structuring during which the problem to be solved is explored and available information is collected. The possible options – responses in terms of the DPSIR framework – are defined and criteria aiming at evaluation of their performance are identified. In the next step the options' performance in terms of the criteria scores is modelled. As a result a matrix – called analysis matrix – is constructed. The analysis matrix contains the raw options' performance with different criteria scales.

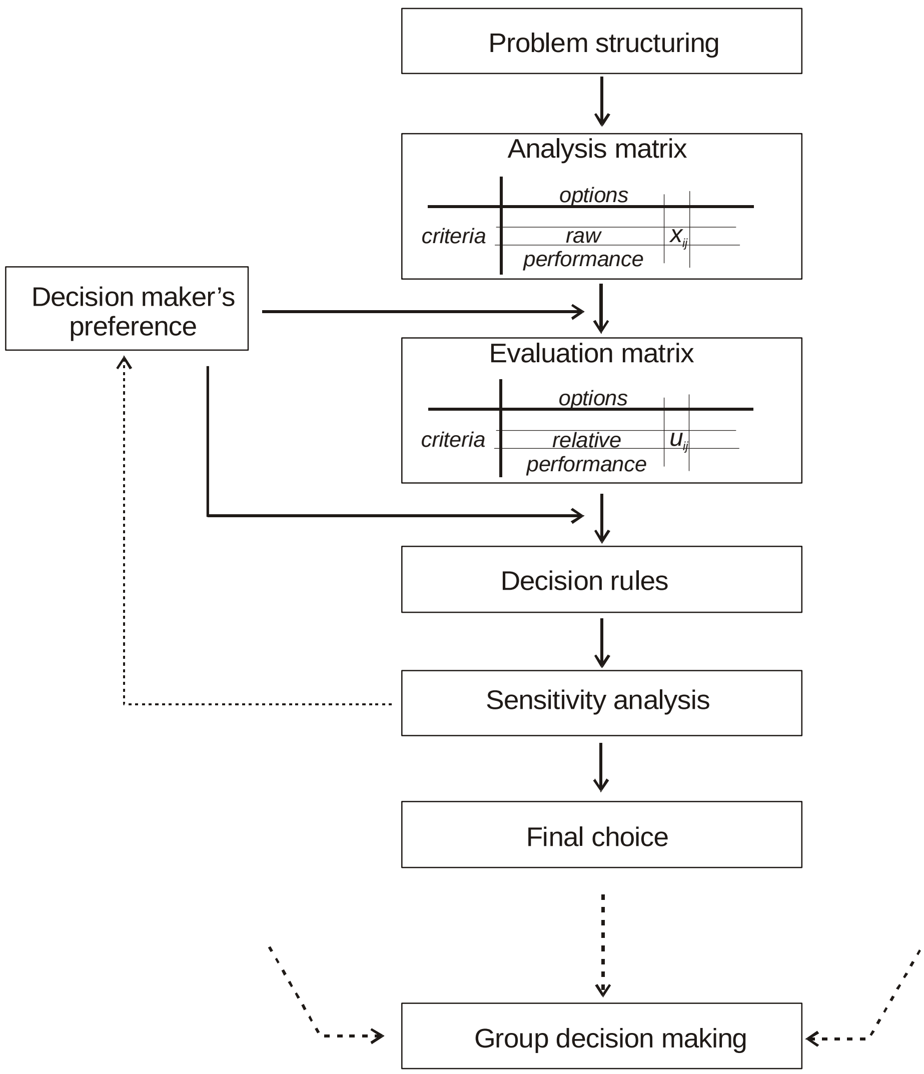

Fig. 1: The basic steps of the MCA that is implemented in the mDSS

Before any aggregation may start, the options' performance with regards to different criteria have to be made comparable. During the normalisation procedures, or at least by applying a value function, the scores are transformed to values on a uniform scale. Since a simple standardisation allows only the transformation of a given value range to a standardised one [0,1], the value function includes human judgements in the mathematical transformation. A value function translates the performance of an option into a value score, which represents the degree to which a decision objective is matched.

Since the main aim of a multicriteria decision analysis is to reduce option of each performances into a single value to facilitate the ranking process, the heart piece of any MCA decision rule is an aggregation procedure. The large quantity of known decision rules differ in the way the multiple options performances are aggregated into a single value. There is no single method that is universally suitable for any kind of decision problem, the decision maker has to choose the method which best corresponds with his purpose. Finally, a sensitivity analysis examines how robust the final choice is to even a small change in the preferences expressed by the decision maker.

In a situation where there are several decision makers involved in the decision process, the individual choices are to be compared and an option is to be chosen, which represents the group compromise decision.

3.2 GENERATING THE ANALYSIS MATRIX

The *analysis matrix* (*M x N: M* options and *N* criteria) is to be built from the environmental indicators identified in the conceptual phase. The cells of the matrix relate to the *option-criterion* pairs and contain the outcomes or consequences for a set of options and a set of evaluation criteria.

In spatial decision-making, the options are a collection of points, lines, and areal objects with associated attributes. The decision outcomes, as in figure 2b-c, may have spatial extensions. For example, in the case of two-dimension a spatial extended decision outcomes (figure 2c), a cell of the decision matrix corresponds to a *map*, which contains the spatially distributed consequences of an option with regards to a criterion. Different to the case of non-dimensional (value- or point-like outcomes, figure 2a) consequences, an additional aggregation must be done.

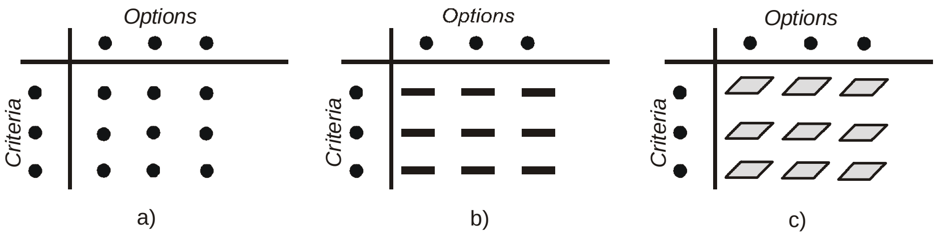

Fig. 2: Different dimensions of decision outcomes: spatial dimension 0 (a); 1 (b) and 2 (c).

3.3 NORMALISING THE ANALYSIS MATRIX

During the standardisation the criterion values expressed in different measurement units are transformed into a common scale, which allows their comparison. The mDSS utilises a linear scale transformation method - the *score range* method. The method doesn't maintain the relative order of magnitude, but scales the raw options' scores precisely in the interval [0,1] (formulas 1-2).

$$x'_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}} \quad \textit{for a criterion to be maximized} \qquad 1.$$

$$x'_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}} \quad \textit{for a criterion to be minimized} \qquad 2.$$

A value x_{ij} corresponds to the option *(i)* and the criterion *(j)*. The notations $x_j^{\min}$ and $x_j^{\max}$ means the lowest and largest score of the *j-th* criterion.

The TOPSIS decision rule (see section 2.4.6) uses vector normalisation (formula 3.). This method has a particular property of producing vectors (the rows of the decision matrix) with the same Euclidean length (equal 1).

$$x'_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} \left(x_{ij}\right)^2}} \qquad 3.$$

3.4 MODELLING VALUE FUNCTION

The value function is another way of transforming the raw criteria scores into a common scale. Hovever, it allows the preferences of the decision maker to be considered during the transormation.

Decision theory provides a theoretical framework for representing the decision maker's preferences about the options' performance. In order to make them more "computational", the preferences are mapped by the value/utility function (*u*). A value/utility[2] function maps the preference about two options *a* and *b* ($a \succ b$: *a* is preferred *b*) in a numerical relation *u(a)* > *u(b)*.

There are several methods for the estimation of the value functions. The mDSS utilises the direct rating method by which the decision maker immediately assigns a value to each criterion score. The shape of the value function may be selected from the implemented set of value functions and only their parameters must be specified. In figure 3 some widely used types of value function are shown.

[2] The term *value function* is used in the context of decision under certainty. The *utility function* refers to the situation under risk consideration, i.e. when the outcomes are associated with a probability.

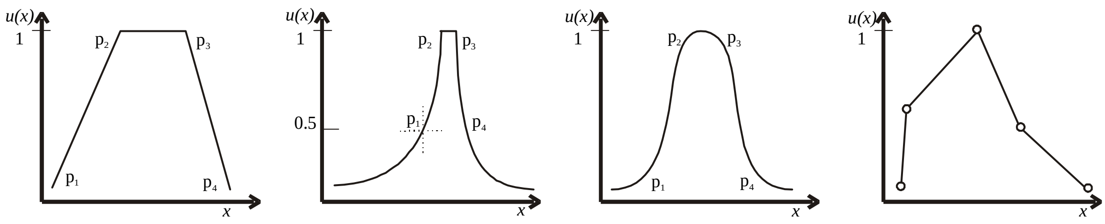

Fig. 3: Some kinds of value function: (a) linear; (b) j-shaped; (c) Sigmoidal: (d) user defined.

The mDSS supports the piece-linear definition of the value function by the user (figure 3d).

3.5 MODELLING CRITERIA WEIGHTS

Decision problems involve criteria of varying importance to decision maker. The criterion weights usually provide the information about the relative importance of the considered criterion. There are many techniques commonly used for assessing the criterion weights such as ranking and rating methods, pairwise comparison and trade-off methods.

Ranking methods use the rank order on the considered criteria. As the rank order describes the importance of the criteria, the information describing them (rank number r_i) is used for generating numerical weights.

$$w_i = \frac{(n - r_i + 1)^p}{\sum_{k=1}^{n}(n - r_k + 1)^p} \qquad 4.$$

n … number of criteria

r_i … rank number of criterion i

p … parameter describing the weights distribution

The parameter p may be estimated by a decision maker through interactive scrolling (as in table 1) or with the help of formula 12 using the weight of the most important criterion as an input from the decision maker. For $p = 0$ results to equal weights. As p increases, the weights distribution becomes steeper. Table 1 shows the estimated weights for some values of p.

		Parameter p						
	Rank	0	0,5	1	2	3	.	10
Most important criterion	1	0,2	0,26	0,33	0,45	0,55	.	0,89
.	2	0,2	0,23	0,26	0,29	0,28	.	0,09
.	3	0,2	0,2	0,2	0,16	0,12	.	0
.	4	0,2	0,16	0,13	0,07	0,03	.	0
Less important criterion	5	0,2	0,11	0,06	0,01	0	.	0
Sum		1	1	1	1	1	.	1

Table 1: The behaviour of the generated numerical weights depending on the parameter p of the rank component method

Pairwise comparison method was developed by SAATY (1980, quoted by MALCZEWSKI 1999) in the context of his decision rule called *Analytic Hierarchy Process*. The method involves pairwise comparisons to create a ratio matrix. Through the normalisation of the pairwise comparison matrix the weights are determined.

The method uses an underling scale with values, from 1 to 9 for example, to describe the relative preferences for two criteria. The result of the pairwise comparisons is a reciprocal quadratic matrix (as in table 2).

1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	…may be used for interpolation between the

I.

	C_1	C_2	C_3	C_4
C_1	1	4	7	5
C_2	1/4	1	1/3	9
C_3	1/7	3	1	5
C_4	1/5	1/9	1/5	1

II.

Table 2: Example of pairwise comparison: (I) Scale for pairwise comparison; (II.) pairwise comparison matrix between 4 criteria (C_1-C_4)

Using the pairwise comparison matrix $A \in \mathbb{R}^{n \times n}$ the weights w_j may be determined as below:

1. Estimate the maximum eigenvalue λ_{max} of the comparison matrix, which fulfil formula 5

$$\det(A - \lambda \times I) = 0 \qquad 5.$$

2. Determine the solution $\tilde{w}$ as in formula 6

$$\begin{aligned} (A - \lambda \times I) \times \tilde{w} &= 0 \\ \tilde{w}_i &\geq 0 \end{aligned} \qquad 6.$$

3. Normalise the $\tilde{w}$ by formula 7

$$w_j = \frac{\tilde{w}_j}{\sum_{i=1}^{n} \tilde{w}_i} \qquad 7.$$

After the weights have been determined, the consistency of pairwise comparison must be evaluated. The procedure of consistency test may be found in Annex 3.1.

The Saaty method deals with the consistency of the pairwise comparison matrix. A consistent matrix means for example if the decision maker says a criterion ***x*** is equally important to another criterion ***y*** (so the comparison matrix will contain value of $a_{xy} = 1 = a_{yx}$), and the criterion ***y*** is absolutely more important as an criterion ***w*** (a_{yw} = 9; a_{wy} = 1/9); then the criterion ***x*** should also be absolutely more important than the criterion ***w*** (a_{xw} = 9; a_{wx} = 1/9). Unfortunately, the decision maker is often not able to express consistent preferences in the case of multiple criteria. Saaty's method measures the inconsistency of the pairwise comparison matrix and sets a consistency threshold which should not be exceeded (for details about consistency of pairwise comparison matrix see the Annex – 4.1).

Swing weights are assigned through the supposed increase of each criterion's performance from the worst to the best value. The criterion which represent the highest boost to the total options' performance is assigned a value of 100, all the other criteria are compared respective to this criterion. As put by Belton and Stewart (2002) put it, "if a swing from worst to best on the most highly weighted criterion is assigned a value of 100, what is the relative value of a swing from worst to best on the second ranked criterion"?

4 DECISION RULES

Decision rules aggregate partial preferences describing individual criteria in a global preference and then rank the options. The decision rules chosen for implementation in the *m*DSS include (i) *simple additive weighting* (SAW), (ii) *order weighting average* (OWA), and (iii) an *ideal point method* (TOPSIS). These decision rules cover a wide range of decision situations and may be chosen by the decision maker according to the specifics of a given decision problem.

- SAW is one of the most popular decision method because of its simplicity. It assumes additive aggregation of decision outcomes, which is controlled by weights expressing the importance of criteria.
- The OWA is being used because of its potential to control the trade-of level between criteria and to consider the risk-behaviour of the decision makers.
- Ideal point methods like TOPSIS order a set of options on the basis of their separation from the ideal solutions. The option that is closest to the ideal positive solution and furthest from the negative ideal solution is the best one.

4.1 SIMPLE ADDITIVE WEIGHTING (SAW)

Simple additive weighting is a popular decision rule because of its simplicity. It uses the additive aggregation of the criteria outcomes (Formula 8).

$$\Phi_{SAW}(a_i) = \sum_{j=1}^{n} w_j \times u_{ij} \qquad w_j \text{ .. criterion weights} \qquad 8.$$

Considering a simple example of two options and three criteria

	w_i	a_1	a_2
c_1	0.4	0.2	0.8
c_2	0.4	0.5	0.11
c_3	0.2	0.9	0.25

The SAW aggregation is performed as following:

$\Phi_{SAW}(a_1) = 0.2 * w_1 + 0.5 * w_2 + 0.9 * w_3 = 0.2 * 0.4 + 0.5 * 0.4 + 0.9 * 0.2 = 0.46$

$\Phi_{SAW}(a_2) = 0.8 * w_1 + 0.11 * w_2 + 0.25 * w_3 = 0.8 * 0.4 + 0.11 * 0.4 + 0.25 * 0.2 = 0.414$

Since $\Phi(a_1) = 0.46 > 0.414 = \Phi(a_2)$, the option a_1 is preferred - $a_1 \succ a_2$

***Example 1**: Aggregation using simple additive weighting decision method*

4.2 ORDER WEIGHTING AVERAGE (OWA)

The Ordered Weighted Averaging (OWA) operator provides continuous fuzzy aggregation operations between the fuzzy intersection (MIN or AND) and union (MAX or OR), with weighted linear combination falling midway in between. It adopts the logic of Yager (1988) and can achieve continuous control over the operator degree of ANDORness and the degree of tradeoff between criteria.

The criteria are weighted on the basis of their rank order rather than their inherent qualities. By so doing the weights – called order weights - are applied to the criteria according to the rank order across their scores. For a given option, the order weight $\boldsymbol{ow_1}$ is assigned to the criterion with the lowest score, order weight $\boldsymbol{ow_2}$ to the criterion with next higher-ranked scores, and so on. Consequently, an order weight $\boldsymbol{ow_i}$ may be assigned to different criteria by two options $\boldsymbol{o_1}$ and $\boldsymbol{o_2}$, depending on the rank order of their scores.

$$\Phi_{OWA}(a_i) = \sum_{k=1}^{n} ow_k \times b_k \qquad 9.$$

where b_k denotes the k-*th* lowest score of the options ***i*** (u_{ij})

Trade-off means that a very low score in one criterion may not be compensated with a very high score in another one. The SAW decision rule described in chapter 2.6.1 allows full tradeoff. The MAXIMAX decision rule, in which the decision maker selects the option with the maximal scores in the best criterion, and the MAXIMIN rule, by which the option with the best scores in the worst criterion is selected, don't allow any tradeoff as the decision is made according only to one criterion.

OWA may be characterized as a control allowing an aggregation between the MAXIMAX, MAXIMIN and SAW extremes. In the case of 3 criteria the set of order weights [1, 0, 0] assigns the extreme importance to the lowest criterion score and corresponds to the MAXIMIN rule. The order weights [0, 0, 1] in contrast assign the extreme importance to the largest criterion score and correspond to the MAXIMAX rule. Equally distributed order weights [0.33; 0.33; 0.33] apply some importance to each rank and don't change the options ranking obtained from the SAW rule.

Considering two options and three criteria as in the table below

	a_1	a_2
c_1	0.2	0.8
c_2	0.5	0.11
c_3	0.9	0.25

The order weights [ow_1 = 0.5; ow_2 = 0.2 ; ow_3 = 0.3] will be assigned to the criteria for each option as following:

	a_1	a_2
c_1	ow_1	ow_3
c_2	ow_2	ow_1
c_3	ow_3	ow_2

The OWA aggregation is performed according to the previous table:

$\Phi(a_1) = 0.2 * ow_1 + 0.5 * ow_2 + 0.9 * ow_3 = 0.2 * 0.5 + 0.5 * 0.2 + 0.9 * 0.3 = 1.4$

$\Phi(a_2) = 0.11 * ow_1 + 0.25 * ow_2 + 0.8 * ow_3 = 0.11 * 0.5 + 0.25 * 0.2 + 0.8 * 0.3 = 0.345$

Since $\Phi(a_1) = 1.4 > 0.345 = \Phi(a_2)$, the option a_1 is preferred - $a_1 \succ a_2$

Example 2: *Aggregation using the order weighting averaging.*

The ANDness, ORness and TRADEOFF characteristics of any particular distribution of the order weights may be calculated using the formulas (10-12).

$$ANDness = (1/(n-1))\sum_{i=1}^{n}((n-i)W_{order\,i}) \quad (10.)$$

$$ORnees = 1 - ANDness \quad (11.)$$

$$TRADEOFF = 1 - \sqrt{\frac{n\sum(W_{order\ i} - 1/n)^2}{n-1}} \quad (12.)$$

n .. number of criteria

i .. criterion rank order

W_{order_i} .. order weight of ***i***-th criterion

In table 3 these characteristics are calculated for a selected set of order weights.

Order weights

	ow1	Ow2	ow3	ANDness	TRADEOFF
MAXIMIN =	1	0	0	1	0
	0,9	0,1	0	0,95	0,15
	0,8	0,2	0	0,90	0,28
	0,5	0,5	0	0,75	0,50
	0,5	0,3	0,2	0,65	0,74
	0	1	0	0,50	0,00
	0	0,8	0,2	0,40	0,28
MAXIMAX =	0	0	1	0,00	0,00
SAW =	0,33	0,33	0,33	0,50	1

Table 3: Characteristics of selected sets of order weights distribution.

Graphical representation of all possible distribution of order weights is shown in figure 4.

Fig. 4: Decision behaviour according to the selected order weight distribution.

4.3 IDEAL POINT METHODS (TOPSIS)

Ideal point methods order a set of options on the basis of their separation from the ideal solution. The ideal solution represents an (not achievable and thus only hypothetical) option that consists in the most desirable level of each criterion across the options under consideration. The option that is closest to the ideal point is the best one. The measurement of separation requires distance metrics. The ideal negative solution may be defined in the same way: the best option in this case is characterised by the maximum distance from it. The formulas 13 and 14 show the generalised definition of distance metrics (using weighted Minkowski L_P metrics). For p = 1, the rectangular or *city distance* is calculated. For p = 2 the Euclidian distance is obtained.

$$s_{i+} = \left[\sum_{j=1}^{n} w_j^p \left(u_{ij} - u_{+j} \right)^p \right]^{1/p} \qquad 13.$$

s_{i+} … separation of the i*th* option from the ideal point
w_j … weight assigned to the criterion j
u_{+j} … ideal value for the j*th* criterion
p … power parameter ranking from 1 to ∞

$$s_{i-} = \left[\sum_{j=1}^{n} w_j^p \left(u_{ij} - u_{-j} \right)^p \right]^{1/p} \qquad 14.$$

s_{i-} … separation of the i*th* option from the negative ideal point
u_{-j} … negative ideal value for the j*th* criterion

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is one of the most popular compromise methods. TOPSIS defines the best option as the one that is closest to the ideal option and farthest away from the negative ideal point. The method requires the cardinal form of the performance of options. The distance from the ideal / negative ideal point is calculated as in Formula 15 and 16.

$$s_{i+} = \left[\sum_{j=1}^{n} \left(u_{ij} - u_{+j} \right)^2 \right]^{0,5} \qquad 15.$$

$$s_{i-} = \left[\sum_{j=1}^{n} \left(u_{ij} - u_{-j} \right)^2 \right]^{0,5} \qquad 16.$$

The relative closeness to the ideal solution (c_{i+}), which will be used for the ranking of options, is calculated as in formula 17.

$$c_{i+} = \frac{s_{i-}}{s_{i+} + s_{i-}} \qquad 17.$$

Considering a simple example of two options and three criteria with already weighted performances

	a_1	a_2	ideal positive solution	ideal negative solution
C_1	0.08	0.32	0.32	0.08
C_2	0.2	0.044	0.2	0.044
C_3	0.18	0.05	0.18	0.05

The distances from the positive and negative ideal solutions as well as the final aggregation according to the formula 17 is performed as following:

	a_1	a_2
s_{i+}	0.24	0.20
s_{i-}	0.20	0.24
c_{i+}	0.46	0.54

For example $s_{i+}(a_1)$ = ((0.08 – 0.32)^2 + (0.2 – 0.2)^2 + (0.18 – 0.18)^2)^0.5 = 0.24

$c_{i+}(a_1)$ = 0.20 / (0.20 + 0.24) = 0.46

Since c_{1+} = 0.46 < 0.54 = c_{2+} the option a_2 is preferred - $a_1 \prec a_2$

Example 3: *Aggregation using the TOPSIS decision methods.*

4.4 ELECTRE

ELECTRE uses a different approach to decision support than value/utility function approaches. It bases on a pairwise comparison of the alternatives, so it's computationally more demanding. It imposes so-called outranking relation on a set of alternatives. An alternative ***a*** outranks an alternative ***b*** if ***a*** is at least as good as ***b*** and there is no strong argument against. There is a variety of ELECTRE techniques, the **ELECTRE III** was implemented in mDSS5. Normally the alternatives have cardinal outcomes in all criteria.

Terms:

i …index labelling a criterion

$z_i(a)$ …. Outcome of the alternative ***a*** with regard to the criterion ***i***

Concordance index [**C(a,b)**] expresses the strength of support, given the available evidence, that ***a*** is at least as good as ***b*** considering all criteria

C_i(a,b) – concordance index over alternative ***a*** and ***b*** with regard to the criterion ***i***

Discordance index [**D(a,b)**] measures strength of the evidence against this hypothesis

D_i(a,b) – discordance index over alternative ***a*** and ***b*** with regard to the criterion ***i***

Preference threshold p_i is a parameter for each criterion *i*

Indifference threshold q_i is a parameter for each criterion *i*

$q_i < p_i$

Veto threshold t_i is a parameter for each criterion i

$\mathbf{w_i}$ weight of the criterion ***i***

The parameters p_i, q_i, t_i is given by user as an input. There is no restriction of the value for these parameters, apart of that in row 26.

Algorithms:

(1) The start point is the analysis matrix. The parameters p_i, q_i and t_i have to be defined by the user.

(2) Compute index $C_i(a,b)$ for all pairs of alternatives *a* and *b*

	$= 1$	if $z_i(a) + q_i(z_i(a)) >= z_i(b)$
	$= 0$	if $z_i(a) + p_i(z_i(a)) <= z_i(b)$
$C_i(a,b)$	$[0, 1]$	if $z_i(a) + q_i(z_i(a)) < z_i(b) < z_i(a) + p_i(z_i(a))$
	$=1- [(z_i(b) - z_i(a) - q_i(z_i(a))/ (p_i-q_i)]$	

(3) Aggregate $C_i(a,b)$ to $C(a,b)$

$C(a,b) = \text{Sum } (w_i \times C_i (a,b)) / \text{Sum } (w_i)$

(4) Calculate a discordance index

	$= 0$	if $z_i(a) + p_i(z_i(a)) >= z_i(b)$
	$= 1$	if $z_i(a) + t_i(z_i(a)) <= z_i(b)$
$D_i(a,b)$	$[0, 1]$	if $z_i(a) + p_i(z_i(a)) < z_i(b) < z_i(a) + t_i(z_i(a))$
	$= (z_i(b) - z_i(a) - p_i(z_i(a))/ (t_i-p_i)$	

Please note, if no veto threshold (ti) is specified, then Di(a,b) = 0 for all pairs of alternatives.

(5) Calculate **Credibility index** $S(a,b)$

	$= C(a,b)$	if $D_i(a,b) <= C(a,b)$ for each ***i***
$S(a,b)$	$C(a,b) \times \Pi((1-D_i(a,b))/ (1-C(a,b)))$	Otherwise; but only for the set of criteria for which $Di(a,b) > C(a,b)$

(6) Determine rank order

Descending distillation

(6.1) Calculate $\lambda max = \max S(a,b)$

(6.2) $\lambda = \lambda max – (0.3 – 0.15\ \lambda max)$

(6.3) For each alternative ***a*** determine the number of alternatives ***b*** with $S(a,b) > \lambda$

(6.4) For each alternative ***a*** determine number of alternatives ***b***

with $(1- (0.3 - 0.15\lambda)) \times S(a,b) > S(b,a)$

(6.5) For each alternative make the difference between (6.3) and (6.4). The alternatives with largest difference (qualification) is called first distillate (D_1).

(6.6) If D_1 has more alternatives than a single one repeat the process on D_1 until all alternatives were classified. If there is a single alternative, than this is the most preferred one. Then continue with the original set of alternatives minus the set D_1, repeating until all alternatives are classified.

Ascending distillation

The same algorithm but in point 6.5 take the options with lowest difference

Final ranking:

There are several ways how to combine both orders. The most frequent is the intersection: Intersection of two outranking relations aRb (a outranks b according to R) if and only if a outranks or is in the same class as b according to the orders corresponding to both relationships.

See for more detail and an application example in Belton and Stewart (2002).

5 SENSITIVITY ANALYSIS

Sensitivity analysis (SA) is an important task in multicriteria decision making: it looks at how robust (or weak) the final decision is, in the case that even a slight change in the decision outcomes or previously expressed preferences is made. Sometimes the sensitivity analysis is distinguished from a robustness analysis: while the sensitivity analysis is assumed as the analysis of the effects of changing data and model parameters in a constrained vicinity to a base solution, the robustness analysis is considered as a systematic analysis of a large set of variations which are plausible in the decision problem context.

5.1 INTRODUCTION

Sensitivity analysis deals with the investigation of potential changes and errors and their impacts on the results of underlying models. Sensitivity analysis, applied post-hoc to decision models, deals with *uncertainties* related to the decision outcomes and/or to the preferential judgements (i.e. value function and criterion weights). The objective is to find out how the options ranking changes by any modification made on the decision models. While the impact of uncertainties on the decision outcomes is mostly analysed by statistical modelling and simulation, the preferential judgements are object of uncertainty during the modelling of weights and value function. However, SA provides neither a explicit probabilistic measure of the risk to make a wrong decision nor an explicit treatment of the risk attitude of the DM.

Sensitivity analysis may be used for wide range of different uses (for detail see Pannell 1997). The SA methods are useful within (i) decision making for identifying critical value/criterion, testing robustness and riskiness of decision; (ii) communication for increasing credibility and confidence; and (iii) modelling process for better understanding of input-output relationship and for understanding the model needs and restrictions.

Sensitivity analysis approaches differ in the level that they address: (i) approaches targeted to local level consider only the immediate neighbourship of a given starting point (e.g. a previously identified optimal solution of a decision problem); while (ii) the approaches working on a global level vary all input parameters over their range of uncertainty. At the same time one or more parameter may be considered to vary. By a single parameter test all other parameters are held fixed. This is a common approach that is used, although the interactions between two or many parameters are ignored and their combined impact is not analysed. On the other side, multi-parameter tests are computationally very complex and thus less practical.

Performing the sensitivity analysis may be a complex undertaking. Considering just a simple example with 5 options and 5 criteria: the simulation of the criterion weight taking into account only 20 different values for a weight (sampling step 0,05) would encompasses 4151 calculations of overall preference function producing 4151 feasible ranking vectors. Additionally, the search for the feasible weights' combination from all 3.200.000 ones is also considerable.

A suitable SA depends on the decision rules chosen for the preferences aggregation. While the decision rules discussed for implementation in the *m*DSS are mostly based on criteria weights, the main concern of the sensitivity analysis will be oriented to the uncertainty addressing the criterion weights.

The *m*DSS utilises two approaches for SA: (i) most critical criterion: identifying the criterion for which the smallest change of current weight may alter the existing ranking of options; and (ii) tornado diagram: graphically comparing the chosen option with any other one and showing ranges within which the parameters may vary.

5.2 MOST CRITICAL CRITERION

This method developed by (Triantaphyllou 2000) considers the most critical criterion to be the one which requires the minimal amount of change in the current value of its weight in order to change the options' rank order. Using this method, the user may directly test if the minimal change in criteria weights which leads to the final rank disturbance is within or outside his confidence range.

The method distinguishes two rank order changes:

- the top critical criterion is that one which changes the best ranked option (T method).
- any critical criterion is that one which changes ranking of any options (A method).

The algorithm to estimate the critical criterion encompasses following steps:

1. List all possible changes in options' rank order caused by modification of criteria weights (e.g. how much must change the weight $\boldsymbol{w_i}$ in order to change the rank order between the first two options). There are only

$$\boldsymbol{n} \times \boldsymbol{m}(\boldsymbol{m}\text{-}1))/2 \qquad 18.$$

changes in rank order possible (with ***n***… number of criteria; and ***m***.. number of options).

In case of three options and two criteria: there are three possible rank order changes which may be caused by changes in two criteria - 2 * 3 * 2 / 2 = **6**

	$\boldsymbol{c}_1$	$\boldsymbol{c}_2$
$\boldsymbol{a}_2 \leftrightarrow \boldsymbol{a}_1$	1	2
$\boldsymbol{a}_3 \leftrightarrow \boldsymbol{a}_1$	3	4
$\boldsymbol{a}_3 \leftrightarrow \boldsymbol{a}_2$	5	6

2. For each change of options' rank order calculate the difference of total options' performance

	ΔP
$a_2 \leftrightarrow a_1$	$(P_2 - P_1)$
$a_3 \leftrightarrow a_1$	$(P_3 - P_1)$
$a_3 \leftrightarrow a_2$	$(P_3 - P_2)$

(P_i is obtained as result of decision rule from the choice phase, aggregation result)

3. For each rank change and for each criterion calculate difference between the options' performance.

	c_1	.	c_n
$a_2 \leftrightarrow a_1$	$a_{21} - a_{11}$	.	$a_{2n} - a_{1n}$
$a_3 \leftrightarrow a_1$	$a_{21} - a_{11}$	.	$a_{3n} - a_{1n}$
. . .	. . .	.	…
$a_m \leftrightarrow a_{m-1}$	$a_{m,1} - a_{m-1,1}$	.	$a_{m-1,n} - a_{m,n}$

(a_{ij} is the corresponding value from the evaluation matrix, i.e. standardised and weighted option's score)

4. Calculate a new matrix dividing the cell values of the matrices created in step 2 and 3.

	c_1	.	c_n
$a_2 \leftrightarrow a_1$	$(P_2 - P_1) / (a_{21} - a_{11})$	.	$(P_2 - P_1) / (a_{2n} - a_{1n})$
$a_3 \leftrightarrow a_1$	$(P_3 - P_1) / (a_{21} - a_{11})$	.	$(P_3 - P_1) / (a_{3n} - a_{1n})$
. . .	. . .	.	…
$a_m \leftrightarrow a_{m-1}$	$(P_m - P_{m-1}) / (a_{m,1} - a_{m-1,1})$	.	$(P_m - P_{m-1}) / (a_{m-1,n} - a_{m,n})$

5. Exclude unfeasible weights changes

The unfeasible solution (no changes in weights are able to change the rank order) should be excluded from consideration. The value of the previous matrix are feasible if there are less then the criteria weights (**w_k**) indicated by user and used for aggregation in the choice phase.

$$(P_j - P_i) / (a_{jk} - a_{ik}) \leq w_k \qquad 19.$$

where ***i*** and ***j*** indicate two options (best option ≤ ***i*** < ***j*** ≤ worst solution) and ***k*** indicate a criterion.

6. The remaining feasible solutions represent changes in the corresponding weights in order to reverse the option ranking. The **rows minimal** (absolute) **value** indicates the critical criterion for a given change in rank and the minimal feasible value in the whole matrix indicates the critical criterion for any changes in options' rank order. To obtain the **critical criterion for top rank** changes the minimal value in the first (***m*** – 1) rows is to be found.

	c_1	.	c_n	
$A_2 \leftrightarrow a_1$	*minimal value indicate critical criterion to reverse the rank order between* $a_2 - a_1$			*minimal value indicate critical criterion to reverse the rank order between the best ranked* (a_1) *and any other option*
$A_3 \leftrightarrow a_1$	*minimal value indicate critical criterion to reverse the rank order between* **any options' pair**			
. . .				
$a_m \leftrightarrow a_{m-1}$				

Considering the following decision matrix (left) producing the aggregated options performances as in the column right

	c_1	c_2	c_2	
w_i	0,33	0,33	0,33	P_i
a_1	0.2	1	0.4	0.528
a_2	0.5	0.3	0.4	0.5*0.33 + 0.3*0.33 + 0.4*0.33= 0.396

After applying a SAW aggregation method the option a_1 is preferred. The goal of the SA is to find out, which criterion weights is most sensible for the final ranking and how much is it to be changed in order to reverse the options ranking.

Calculation of the differences in options performances (a_{2i} - a_{1i}) - left; right - calculation of difference in options' performance (ΔP)

	c_1	c_2	c_3	ΔP
$a_2 \leftrightarrow a_1$	0.5 – 0.2 =0.3	0.3 - 1 = - 0.7	0.4 – 0.4 = 0	0.396 - 0.528 = - 0.132

Calculation of the (P2 - P1) / (a2i - a1i)

	c_1	c_2	c_3
$a_2 \leftrightarrow a_1$	- 0,132/0.3 = - 0.44	- 0,132/ - 0.7 = 0.188	∅

Identifying the most critical criterion and the magnitude of the change of its weight

	c_1	c_2	c_3
$a_2 \leftrightarrow a_1$	- 0.44	0.19	∅

➔ ***Consistency check*** – change of both criteria weights may cause rank reverse because both value above are less then the current criteria weights

➔ ***Critical criterion*** = c_2 because |0.188| < |-0.44|

Calculation of new weight w1'

$w_{2'}$ = 0.33 - (0.19) = 0.14

Standardisation of new weights (w*)

w1* =w1/(w1'+w2+w3)=0.33/0.8 = 0.41

w2* =w2'/ /(w1'+w2+w3) = 0.14/0.8 = 0.18

w3* = w3/(w1'+w2+w3) = 0.33/0.8 = 0.41

Proof: Applying the new weight set the resulting aggregated performances are equal.

	c_1	c_2	C_2	
w_i	0,41	0,18	0,41	P_i
a_1	0.2	1	0.4	0.42
a_2	0.5	0.3	0.4	0.42

Example 4: *Most critical criterion approach for sensitivity analysis applied for the SAW decision rule.*

5.3 TORNADO DIAGRAM

The tornado diagram is a graphical method of sensitivity analysis. The advantage of this approach is its visual representation of sensitivity that compares two options (a basic and a challenging one) at a time. The horizontal bars represent the ranges of the options' total performance obtained by the variation of each weight. Bars are arranged from widest to narrowest and thus produce a "tornado" shape (figure 5).

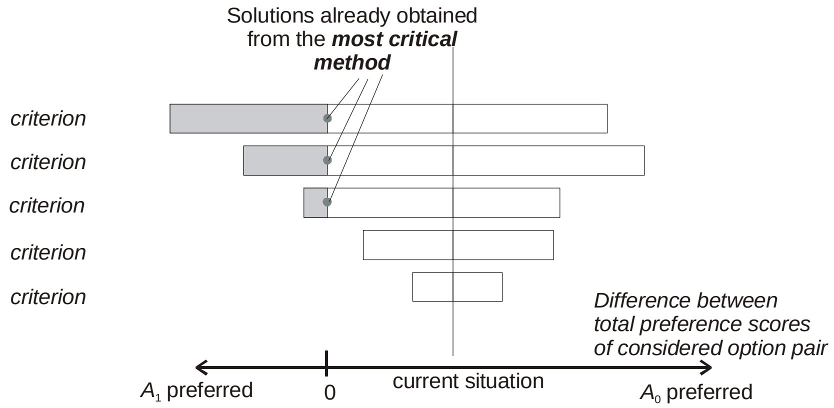

Fig 5: The tornado diagram showing the differences in total performance of two options obtained by varying of criteria weights.

On the x - axis the difference in total performance between compared solution is shown. Notice that the zero points – with equal performance of both options – correspond to the weights obtained from the critical criterion approach.

6 GROUP DECISION MAKING - AGGREGATION OF PREFERENCES

Decision making which involves two or more decision makers encompasses a variety of techniques, which allow the group to search for solutions and to evaluate them. Decision theory distinguishes two main streams in these cases: (i) group decision making characterised by the common effort of a group of decision makers to resolve a given decision problem; and (ii) game theory, which also handles situations where two or more decision makers (players) face a decision problem, but in this case each player wants his own optimal solution to be accepted.

6.1 INTRODUCTION

Group decision making regards a situation in which two or more decision makers are involved in a joint decision whereas each of them has his own perception of the decision problem and the decision consequences. According to (Choi; Suh, and Suh 1994) group decision problems are social problems rather than mathematical ones with only few methodologies to verify their fairness, i.e. the way in which the individual preferences are aggregated. Various attempts have been undertaken to extend MCA techniques to be able to deal with interpersonal conflicts. The different preferences of decision group members create a new “dimension” of a decision problem, which, in order to obtain a common decision model, has to be aggregated in a similar way as the preferences for multiple criteria are dealt with in MCA.

“Behaviour aggregation” is the name for the process by which the group members are able to compromise their expectations and agree on a common system of objectives and preferences. After an communicative phase, the decision makers assume a unified problem structure and common value/utility functions.

If this process fails – i.e. the behaviour of any group member is uncooperative – formal aggregation procedures (voting rules) may be used to select a compromise solution. In this case each decision maker may solve the given decision problem on his own. The individually chosen solutions are then presented and compared to each other through voting. A large decision group may take advantage of this procedure ,but in a small group there is a risk, that one group member (dictator) systematically affects the decision process and thus “dictate” a solution.

6.2 COMPROMISING CRITERIA WEIGHTS

If the *m*DSS were to be used in a group decision situation, first the behaviour aggregation approach might be proposed. The decision group members are asked to discuss the problem structure, the main goals and their concrete operative attributes, and the decision preferences expressed through the value functions and criterion weights. If such a co-operative approach is possible and the decision group reaches an agreement, the process would require only an advanced exploration technique for the table and spatial data views. The function of supporting communication within the group does not necessary suppose that all members interact at the same place and time.

Should the problem understanding and the preferences be slightly different, but the group members are principally willing to make a common solution, some procedures that compromise the differences may be implemented. The group members may have different expectations regarding the considered criteria and options, the value/utility function that is used and the importance of criterion (weights). If the differences are small, a mathematical approach or an approach dealing with incomplete information may be chosen. If the differences are not overcome, then a voting rule or an inter-personal preference comparison approach should be chosen.

6.3 COMPROMISING THE FINAL SOLUTION

In this case the weights are to different or the group was not able to compromise to find a common value function for all criteria. In order to find a compromise solution, the final ranking is to be taken into consideration. The Borda technique assigns ranks to options based on the rationale that the higher the position of an option on the voter's list, the higher the rank assigned. The voting position of an option is determined by adding the ranks for each option from every voter using the Borda vote aggregation function. The winner is an option that receives the highest score calculated such that all options are assigned a score starting with 0 for the least favourable solution, 1 for the second worst, 2 for the third worst, and so on. All scores are weighted by the number of voters, resulting in the Borda score for each option.

Borda vote aggregation prevents a contentious option that ranks very high with some group members and very low with others from winning, and promotes a consensus option. Low variance scores are indicative of situations where the decision-makers prioritised an option in about the same position in the list in each of their individual lists ("0" indicates exactly the same position for all three). High variance scores are indicative of one decision maker ranking an option higher in a ranked-list relative to another decision maker who might have ranked the same site lower, or in the middle of the list.

6.3.1 INDIVIDUAL RANKING

Each decision group member performs the decision analysis on his own (using his own copy of the mDSS). The result of each individual decision is supposed to be a full ranking of all available options.

6.3.2 GROUP RANKING – BORDA RULE

Group decision making compromises the individual rank orders of options. Each group member attributes an individual Borda mark to each option. The borda mark shows an option's preferential relationship to other possible options (formula 1). The best option in a individual ranking obtains (***n***-1) value, where ***n*** is number of criteria. Similarly, the worse option in a given ranking is marked with 0.

$\succsim_k$ preference relation of decision maker ***k***

$a_j \succsim_k a_i$ options $\boldsymbol{a_j}$ is preferred by the decision maker ***k*** to option $\boldsymbol{a_j}$

$r\left(a_j \mid A, \succsim_k\right)$ .. number of options that decision maker ***k*** ranks at most as good as $\boldsymbol{a_j}$ (number of options which are less ranked as $\boldsymbol{a_j}$)

$$r\left(a_j \mid A, \succsim_k\right) = \# a_i \in A \mid a_j \succsim a_i \qquad 20.$$

To determine the consensus ranking, the total Borda mark is calculated according to formula 20. The individual marks are summarised for each option and the best (consensus) option is the one with highest total Borda mark.

A option $\boldsymbol{a_j}$ is preferred another option $\boldsymbol{a_j}$ ($a_j \succsim a_i$) in the final group ranking only if

$$\sum_{k=1}^{m} r\left(a_j \mid A, \succsim_k\right) \geq \sum_{k=1}^{m} r\left(a_i \mid A, \succsim_k\right) \qquad 21.$$

Considering a simple example of three decision makers {A,B,C} and three options {a_1, a_2, a_3}

A) Initial individual rankings

	Best options	..	Worst option
Decision maker A	a_1	a_2	a_3
Decision maker B	a_2	a_1	a_3
Decision maker C	a_2	a_3	a_1

B) Individual Borda mark

	Option a_1	a_2	Option a_3
Decision maker A	2	1	0
Decision maker B	1	2	0
Decision maker C	0	2	1

C) Total Borda mark

	Option a_1	a_2	a_3
Total Borda mark	3	5	1

D) Final ranking

Since the highest Borda mark has been assigned to the option a_2 and the second highest to the option a_1, the group ranking is $a_2 \succ a_1 \succ a_3$

Example 5: *Group Ranking – Borda rule.*

6.3.3 ALTERNATIVE GROUP RANKING

In the mDSS5 a few alternative group decision algorithms have been implemented. **Condorcet winner** (called after the mathematician and philosopher Marie Jean Antoine Nicolas Caritat, the Marquis de Condorcet) is an option which, when compared individually with each of the alternative options, is preferred by a majority of voters. The Condorcet winner is estimated by a series of imaginary pairwise contests. In each contest, the option wins which is ranked higher than the other one by most voters (decision makers, stakeholders). When all possible pairs of options have been considered, the option which beats every other options in these contests is declared the Condorcet winner. Due to be cyclic collective preferences, there may be no Condorcet winner. In such a case other group decision method need to be applied.

The second algorithm for group decision making added to mDSS5 resembles the OWA rule. Following this methods, a new set of weights is assigned to different rank positions. The weight vectors are assumed to be decreasing with highest weight being assigned to the top position of the option ranks. Depending on the actual values of the weights, several strategies can be distinguished:

Plurality voting: (1,0,0,…,0): All rank positions but the first are given weight 0. The winner is the alternative that is placed at the first position in the orders by the greatest number of experts.
Anti-plurality voting (1,1,1,…,0): All rank positions but the latest are given weight 1. In this method the experts indirectly point out the alternative judged as the worse one. The winner is the alternative that is placed at the last position in the orders by the least number of experts.
Z–plurality method (1,1,….,n-z = 1, 0,0,…0): The first n-z rank positions are assigned the weight 1 (n .. number of options, z…arbitrary chosen number, n>z). It is easy to show that if z=n-1, plurality voting is produced, and if z=1, the corresponding results equal to anti-plurality strategy.

To calculate collective preference, the weights are multiplied with the number of times an option was ranked at a certain position.

7 ANNEX

7.1 CONSISTENCY TEST OF RECIPROCAL MATRIX OF PAIRWISE COMPARISON

The important point is that the expected rank of ***A*** (if ***A*** is consistent) is 1 and the expected *eigenvalue* is equal to number of compared criteria (n). In inconsistent cases the expected maximum eigenvalue is grater than n while the others are close to zero. The *eigenvector* of matrix ***A*** is an estimate of the relative weights of the criteria being compared.

In ideal cases the comparison matrix (***A***) is fully consistent, the rank(***A***) = 1 and $\lambda = \boldsymbol{n}$ (***n*** = number of criteria). In this case, the following equation is valid:

$\boldsymbol{A} \times \boldsymbol{x} = \boldsymbol{n} \times \boldsymbol{x}$ (where ***x*** is the *eigenvector* of ***A***)

In the inconsistent' cases (which are more common) the comparison matrix ***A*** may be considered as a perturbation of the previous consistent case. When the entries a_{ij} changes only slightly, then the *eigenvalues* change in a similar fashion. Moreover, the maximum *eigenvalue* (λ_{max}) is slightly grater to n while the remaining (possible) *eigenvalues* are close to zero. Thus in order to find weights, we are looking for the *eigenvector* which corresponds to the maximum *eigenvalue* (λ_{max}).

The consistency index (CI) is calculated as following

$$CI = \frac{\lambda_{\max} - n}{n-1} \qquad 22.$$

Then, the consistence ratio (CR) is calculated as the ratio of consistency index and random consistency index (RI). The RI is the random index representing the consistency of a randomly generated pairwise comparison matrix It is derived as an average random consistency index (Table 4) calculated from a sample of 500 of randomly generated matrices based on the AHP scale.

$$CR(A) = \frac{CI(A)}{RI(n)} \qquad 23.$$

If CR(A) ≤ 0.1, the pairwise comparison matrix is considered to be consistent enough. In the case CR(A) ≥ 0.1, the comparison matrix should be improved. The value of RI depends on the number criteria being compared.

n	1	2	3	4	5	6	7	8	9
RCI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45

Table 4: Random consistency indices for different number of criteria (n).

8 GLOSSARY

The **DPSIR framework** shows a chain of causes-effects from Driving forces (activities) to Pressures, to changes on the State of environment, to Impacts and Responses. DPSIR is based on the assumption that economic activities and society's behaviour affect environmental quality. The relationships between these phenomena can be complex. DPSIR highlights the connection between the causes of environmental problems, their impacts and the society's response to them, in an integrated way.

- The ***Driving forces*** are represented by natural and social processes which are the underlying causes and origins of pressures on the environment. E.g. agriculture/land use change, industry, waste management. The ***Pressures*** are outcomes of the driving forces, which influence the current environmental state. They are the variables which directly cause (or may cause) environmental problems. E.g. polluting emissions, noise.
- The ***State*** describes physical, chemical or biological phenomena in the given reference area. It reflects the condition of the environment. E.g. air, water, soil quality.
- ***Impacts*** on population, economy, ecosystems describe the ultimate effects of changes of state, in terms of damage caused. E.g. eutrophication, biodiversity loss.
- The ***Responses*** demonstrate the efforts of society (e.g. politicians, decision-makers) to solve the problems. E.g. policy measures.

From the point of view of the decisional context, the Impact describes the existing problem. The negative Impact arises as a change in the environment's State reduces the value (either in quantitative or qualitative terms) of the natural resource. The Response refers to the decision act: the chosen option aimed at reducing the negative pressures on the state of the environment. Driving forces, Pressures and States are the possible levels of intervention: a decision maker can choose one of them (or a combination of them) as a concrete object for his response.

Multi-criteria analysis or multi-criteria decision aid (MCA) is a well known branch of decision theory, sometimes considered as a part of Operational Research, dealing with a number of considered evaluation criteria. Multi-criteria analysis can be used to describe any structured approach to determine overall preferences among alternative options, where the options accomplish several objectives.

A **decision problem** is considered to exist, when a planner or decision maker (DM) perceives a discrepancy between the current and desired states of the planning system, and when (i) the DM has alternative courses of action available; (ii) the choice of action can have a significant effect on this perceived difference; and (iii) DM is motivated to make a decision, but he or she is uncertain a-priori as to which option should be selected.

An **indicator** can be defined as a parameter or value derived from parameters, which provides information about a phenomenon. In particular, an environmental indicator is a parameter, which provides information about the situation or trends in the state of the environment, in human activities that affect or are affected by the environment, or about relationships among such variables.

In Multiple Criteria Analysis, it is an instrument which allows for the synthesis of certain information to lay the foundation for judging an action. This synthesis can be relevant in qualitative or quantitative terms, and relative to particular characteristics, attributes or effects (consequences) which might arise from the action's implementation. Several indicators may be synthesised to define a criterion encompassing a broader point of view.

An **index** is a set of aggregated or weighted parameters or indicators.

Options/possible solutions/ course of actions represent decision acts whose outcomes can be simulated. Possible solutions comprise all feasible actions or activities that solve or contribute to the solution of the decision problem. Relating to the DPSIR framework, they represent the possible responses proposed to address the impacts.

Criterion is a measure against which options are assessed to evaluate the degree to which they achieve objectives. A tool for evaluating and comparing the potential actions according to a well-defined point of view.

Value function is a mathematical representation of human judgements. It translates the performances of the options into value scores, which represent the degree to which a decision objective is matched.

Decision rule is the procedure by which the relative outcomes of available options are aggregated. Through a decision rule the multi-dimensional description of an option's outcomes which refer to a set of decision criteria is transformed into the single value of an overall option's performance.

Evaluation is a process of examining options and assessing their relative performance with regard to the selected criteria. In MCA the evaluation process encompasses both the assessment of the options' outcomes (by means of value function) as well their aggregation by decision rules.

Alternative scenarios are hypothetical future events. They establish the social, environmental and socio-economic settings that can create changes in driving forces, when human activities are involved, and in state, when dealing with the environment. It is an exploration of a possible future for which an underlying set of assumptions has been made.

End user is the person that will use the *m*DSS to examine alternative strategies in water management in the catchment. End users are essentially institutional decision-makers who could use the results of the project in their activity as water managers. To facilitate communication and to avoid misunderstandings, they can be called "DSS users".

Stakeholder is a social actor (individual or collective), who is an actual or a potential user of water resources for different purposes such as agriculture, industry, domestic consumption, recreational, or communication. Stakeholders are affected by the decisions of DSS users.

Integrated assessment (IA) is an interdisciplinary process of combining, interpreting, and communicating knowledge from diverse scientific disciplines in such a way that the whole set of cause-effect interactions of a problem can be viewed from a synoptic perspective (Rotmans and Dowlatabadi, 1998). Integrated assessment implies that science is exemplary and that it is being done in the context of social and economic forces at work in society. It is a new kind of science coupled to new economics, new sociology and new management policies (Harris, 2002).

Integrated assessment and modelling (IAM): is an interdisciplinary and participatory process combining, and interpreting communication knowledge from diverse scientific disciplines to allow a better understanding of complex phenomena (Rotmans and Van Asselt, 1996) The main purpose of IAM should be to inform policy and to support decision making. In some cases, when IAM is difficult or impossible to achieve, the process of IAM rather than the outcome allows important lessons to be learnt. IAM can be a methodology used for gaining insight over environmental problems, a framework to organise disciplinary research, a tool to integrate insights from the natural and social sciences.

IAM seeks to achieve multiple forms of integration in its approach to environmental issues (integration = linking models with GIS, integrating software, integrating stakeholder participation, integrating different scales, disciplines and models).

Today IAM combines the natural and social sciences to provide a broader view of the system and the impediments to better management and sustainability. It also seeks to enhance communication both between researches and stakeholders, and among IAM participants. It is a problem-focused area of research with studies often undertaken on a demand-pull, or stakeholder needs basis.

Tomorrow (optimistically) IAM tools may successfully integrate insights from the natural and social sciences. The results from the modelling and testing of alternate future scenarios are used to depict scenarios that extend beyond the recent range of experience to assist in the evaluation of more extreme events or frequencies and to develop appropriate environment and resource management policies. These results are effectively communicated to politicians, decision-makers, and community partners who then use the findings in their decisions about future actions. The overall pattern is one of integrated modelling, integrated application, integrated communication and integrated decision making (Parker et al., 2002).

Multi-sectoral: the components of a natural and/or social system; a particular resource or segment of the economy.

Integrated water resources management includes the planning and management of water resources and land. This takes account of social, economic and environmental factors and integrates surface water, groundwater and the ecosystems through which they flow.

Task Force Group

Within the NetSyMoD framework, the Task Force Group (TFG) will have the role of overseeing and steering the whole process. The TFG will include insiders – that is, actors directly involved in the process, familiar with it, or with specific expertise of relevance – and outsiders – people who are not familiar with the issue, but who can provide more objectivity, as well as fresh perspectives, mitigating the potential biases emerging from insiders' pre-existing relationship with experts and stakeholders. The usefulness of involving outsiders in the TFG is limited in the case of experts' consultation, and may thus be omitted without the risk of biasing the process. One or more facilitator(s) are then needed to support the TFG, and analyse the outcomes of the brainstorming exercise. The facilitator's role is crucial for providing a correct and effective management of participation, even in these early stages. For the sake of efficiency, it is suggested that the TFG should include between 4 and 8 members.

Brainstorming meeting:

Loosely defined, any group activity involving the pursuit of new ideas can be defined as a brainstorming meeting. Thus, the purpose of a brainstorming meeting is to produce new ideas about a specific topic. A facilitator should be appointed to control the flow of information, and a member of the team to record ideas and report them back to the meetings' participants.

Link to: http://www.scottberkun.com/essays/essay34.htm

Snowball sampling technique:

The snowball technique is used for identifying hidden population such as groups whose organisational capacity is limited and who may not be easily recognisable. The sampling process begins with the TFG identifying the "seeds", a relatively small number of people who are the first to be involved in the process. These seeds are then asked to name other actors belonging, in their view, to the same group of interested parties.

Social Network Analysis:

Social Network Analysis (SNA) is a framework strategy for investigating social structures, which enables researchers to translate core concepts of social and behavioural theories into a formal language, based on relational terms. Wetherell et al., 1994 define SNA in the following way:

"Most broadly, social network analysis (1) conceptualises social structures as a network with ties connecting members and channelling resources, (2) focuses on the characteristics of ties rather than on the characteristics of the individual members, and (3) views communities as ‘personal communities’, that is, as networks of individual relations that people foster, maintain, and use in the course of their daily lives.” (p. 645)

SNA provides procedures to determine how a social system behaves, and mathematical and statistical methods to test the validity of theoretical underlying hypotheses of human behaviour and interactions.

In SNA, actors can be individuals, groups, corporations, etc., who are interdependent. Relational ties establish a linkage between pairs of actors: relations may, for instance, express the evaluation of actors with respect to one another, or they may quantify transfer of resources between actors; there may be behavioural interactions between actors, or physical connections. Relations may be directed (e.g. actor A phones to actor B) or undirected/reciprocal (the existence of a specific relation between actor A and B implies the same relations between B and A). Ties may either be present or absent, or they may have different strengths/values associated with them, etc. Actors and relations together form networks: networks, therefore, are the results of a process of defining a group of actors on which ties are to be measured.

Questionnaire:

Questionnaires should be used when the respondents can answer the questions directly, or when the respondent is a representative of an entity. In addition, questionnaires can also be used to structure face-to-face or telephone interviews. Within the NetSyMoD framework, questionnaires administered either through face-to-face interviews or mail surveys are suggested.

Within the proposed framework, the questionnaire should include at least three sections:

1. Stakeholders’ identification.
2. Stakeholders’ relations.
3. Stakeholders’ views of the problem.

Stakeholders’ identification: key attributes of the person interviewed should be recorded in this section, including affiliation and role.

Stakeholders’ relations.

The interviewees will be asked to (i) identify the actors whom they interact with; and (ii) to identify the type of relationships existing with each of the actors mentioned previously, as well as the frequency.

Stakeholders’ views of the problem.

Information on stakeholders’ understanding of the specific decision-making problem, as well as their preferences. Part 3 of the questionnaire is clearly highly specific to the problem being analysed, and it should be structured with open-ended or semi-structured questions. However, it should include information regarding both the problems, and the preferred management responses.

Interviews

Telephone or face-to-face interviews are often used to gather data on egocentric networks. With this technique, there is a need to identify the right line of questioning, and minimise interviewer bias by providing a standard checklist that should be followed. Open-ended questions should be preferred, as they will provide more information (although at the cost of increased difficulty in codifying the data, and comparing across actors).

Key actors

Through a positional analysis, key actors will be identified. Actors are structurally equivalent if they have identical ties to and from all other actors, and on all types of relations – structurally equivalent actors are, therefore, substitutable and, if two or more actors are structurally equivalent, there is no loss in generality in aggregating them. For the purpose of NetSyMoD, positional analysis should be carried out both on the basis of relations and views of the problem. A “similarity threshold” needs to be established, which will determine the degree of similarity required for actors to be considered substitutable.

Power structure

The distribution of power determines the synergies and interactions emerging in the network. The researcher – or the policy maker – will assess the strength and direction of identified relations, and single out those actors who are in a “central” position in the network – that is, those who play a crucial role, and to whose opinion/position the researcher/decision maker needs to pay particular attention to.

Central:

Traditionally, centrality measures of actors have been considered as good proxies for power position, based Freeman’s measures (Freeman, 1979):

- Degree centrality is the number of direct ties that involve a given node; it represents the level of communication activity – or the ability to communicate directly with others
- Closeness centrality depends on the minimal length of an indirect path between non-adjacent nodes, and represents independence – or the ability to reach a large numbers of alters while being able to rely on a minimum number of intermediaries
- Betweenness centrality reflects the intermediary location of a node along indirect relationships linking other nodes. A node with high betweenness has the capacity to facilitate or limit interactions between the nodes that it links. This measure represents control over communication – or the ability to restrict communication of others.

Role

The concept of role explores the behaviour expected of a person occupying a particular social position.

Position

The position occupied by individual actors within the network is intended as the space in the network defined by the way in which occupants of a certain position relate to actors in other positions. Thus, the concept of social position refers to a collection of actors embedded in similar ways in the network.

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